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The Platonic Foundations of Finance and the Interpretation of Finance Models

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What is the nature of the collection of mathematical models we call "finance?" There is knowledge in finance, but what is the nature of it, and what is it really about? What sorts of justified true beliefs do we have, i.e., what would it mean for these beliefs/models to be true, and how do we justify their truth? The traditional positive interpretation of finance models poses a number of difficult, and perhaps even intractable, philosophical problems. There are as many as six other interpretations, only one of which, the normative interpretation, is familiar. Most of finance's interpretations of its models, including both of the usual ones, fall within the platonic/foundational perspective as it is applied to mathematics. Even the positive interpretation, to which most in finance implicitly or explicitly subscribe, does not reveal the "objective reality" that is supposed to be out there. We clearly need a truly "behavioral" finance, in every sense of the word.

Introduction: What is Finance?

It is common to define "finance" (AD—the Academic Discipline[1]) as the science of "finance" (SSP—the Social Structures and Processes concerning the storage and transfer of monetary resources).[2] All sciences concern knowledge, and knowledge is often defined as "justified true belief." What distinguishes scientific knowledge from other forms is the nature of its beliefs and the methods by which they are justified.

Science is interested in explanations, the answers to "Why?" questions, and finance is interested in explanations that permit predictions, the answers to "What will?" questions. It follows then that "true beliefs" are predictions that correspond with what actually happens, and scientific "justification" is a formal process of making predictions and testing whether they "come true" according to a correspondence theory of truth. This is how finance claims to work. "Theoretical finance" makes predictions, and "empirical finance" tests their validity. In principle, it is a simple process in which the "facts" (what happens) are supposed to speak for themselves.

In practice, however, it is not so simple. To begin, consider the forms in which finance casts the beliefs it thinks it has justified as true or hopes to justify as true. Where in finance do we look for knowledge, and what does that knowledge look like? Finance has "models" (e.g., discounted cash flow models, Modigliani and Miller models, asset pricing models, option pricing models, stochastic models), "hypotheses" (efficient markets hypothesis), and "laws" (the law of one price). It also has theories (modern finance theory, modern portfolio theory, capital structure theory, dividend theory, agency theory, signaling theory, asymmetric information theory, arbitrage pricing theory, option pricing theory). There are no explicit definitions in finance of these terms, and this is not unusual.

Philosophers usually contrast "theory" with "observation," which parallels the contrast between "theoretical" and "empirical" in finance. Scientists, however, see "theories" as structuring the subject matter (Sismondo [1996]), and this is consistent with the conventional use of the term in finance. From the familiar examples in the last paragraph, we might infer that a "theory" is a set of related statements that are supposed to explain something, or to answer a "Why" question. A "model" is one of these statements in the form of a mathematical equation.

For example, the Black-Scholes option pricing model and the binomial option pricing model are parts of option pricing theory. The capital asset pricing model is part of modern portfolio theory, and the Modigliani and Miller models are parts of capital structure theory and dividend theory.[3]

And there are different sorts of "theories." Capital structure theory and dividend theory are sets of possible explanations for corporate policies. Agency theory, signaling theory, and asymmetric information theory, which are essentially descriptions of behaviors that are thought to explain financial phenomena, are parts of both. All are parts of modern finance theory. So a "theory" can be either a set of answers to a single "Why"

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question, a single answer to a set of "Why" questions, or a set of answers to a set of "Why" questions.

"Hypothesis" and "law" are used much less frequently in finance as proper nouns, which makes it difficult to infer any conventions from their use. Based on a single example (efficient markets hypothesis), a "hypothesis" can be considered the non-mathematical counterpart of a model. To formally test a hypothesis requires a mathematical model like the market model or the random walk model.

And also based on a single example (law of one price), a "law" can be considered a statement in which we unconditionally believe.[4] So "laws" *are* true, or at least we believe they are true. "Models" and "hypotheses" *may* or *may not* be true, or at least we believe we can tell the difference. But what about the truth of theories?

Modern finance theory and modern portfolio theory are sets of statements united by a common methodology or set of methods; in other words, they are Kuhnian paradigms. These theories are true if they reflect paradigms capable of generating true statements, regardless of whether any of their current statements are true. In effect, they are true or false approaches to truth. Therefore, modern finance theory and modern portfolio theory can be true even if none of their statements are. Capital structure theory, dividend theory, and option pricing theory are sets of statements about capital structure policy, dividend policy, and option pricing, respectively. These theories incorporate other theories and models, which may or may not be true.

We might say that capital structure theory, dividend theory, and option pricing theory are true if one or more of their statements is true. And even if they do not incorporate any true statements now, this does not preclude them from being true in the future when they do. Agency theory, signaling theory, and asymmetric information theory define "agency," "asymmetric information," and "signaling." In this sense, they are necessarily true. But as theories within other theories, such as capital structure theory and dividend theory, they may or may not be true.[5]

For example, modern finance theory is a way of doing financial research. It is true if it works at producing knowledge (justified true beliefs or true statements) about finance. At present, we believe it is working. Capital structure theory is a part of modern finance theory. At present, we do not know if it is true, since we cannot now explain why firms have chosen their capital structures (or at least not in the way that modern finance theory construes "explanation"). But what motivates our research is the belief that it will someday be considered true; that is, we believe we will find an explanation. Signaling theory is one possible explanation for capital structure. At present, we believe there is a signaling phenomenon, but we do not know whether it explains capital structure.

Finance "theories," as we conventionally use that term, are not themselves capable of truth or falsity. Rather, they are or are not fruitful approaches to truth (e.g., modern finance theory), may or may not incorporate truth now or in the future (e.g., capital structure theory), and are or are not true or false in different applications at the same time (e.g., signaling theory).

Finance theories are sets of mathematical statements ("models") or non-mathematical statements ("hypotheses") that *are* thought to be capable of truth or falsity. A finance "theory" is just a way of structuring, organizing, or referring collectively to finance models and hypotheses, and, as we have shown, the term "theory" can be used differently and somewhat inconsistently. We cannot test a finance "hypothesis" for truth or falsity without the use of a "model."

So after we sift through all of finance, the "models" are where we should look for the science, since these are the "beliefs" whose "truth" we seek to "justify." "Theoretical finance" is model-building, and, as we have asserted, all the "models" in finance are mathematical. This use of mathematics in finance, despite its "objective" appearance, is more a belief reflective of our values.[6]

On the one hand, the use of mathematics would seem to facilitate "empirical finance," since the explicitness of mathematical equations should make it easy to determine correspondence. Something either is or is not equal to something else. For example, it is easy, in principle, to test the stochastic process-based Black-Scholes option pricing model. Observed prices either are or are not equal to the prices predicted by the model from the observed variables. But it may not be valid to subject a culturally based hypothesis such as "Options prices are a consequence of investors following financial fashion trends" to traditional hypothesis testing. The hypothesis may still be valuable in understanding the option pricing phenomenon, however.

On the other hand, the use of mathematics may complicate "empirical finance," since mathematical equations are not the same as the social structures they are supposed to represent. They are not just simplifications or abstractions. Although we may find that something is equal to something else, that is, the model is "true," it is not at all clear what that has to do with the "truth" of what it is that has been modeled. It is our belief in the appropriateness of a mathematical approach and not any a priori or even a posterior evidence of its superiority that has caused us to choose such methods.

This paper considers the nature of mathematical models we call "finance." There is knowledge in finance, but what is the nature of this knowledge? And what is it really about? What sorts of justified true beliefs do we have? In other words, what would it mean for these beliefs/models to be true, and how do we justify their truth?

These are admittedly very abstract questions that can only be clarified as we begin to answer them. For a more concrete example, consider the Black-Scholes option pricing model. Is it about options in real markets? What is and is not an option? Is it really a "justified true belief" if we find that it predicts options prices, or should we be looking for "something more" that explains why its predictions are true? Are there other ways it can be true that do not concern prediction?

Since finance models are all mathematical, to understand what a model *is* and what it means for a model to be true, it is necessary to understand what mathematics *is*. This understanding enables us to suggest possible answers to the question of what finance *is*. Traditionally, finance believes in a *positive* interpretation of its models (i.e., they represent things as they are), and we first address this claim. But this interpretation has problems, and there are a number of alternative interpretations for which equally strong and perhaps even stronger arguments can be made.

We conclude that for a discipline that believes itself to be firmly grounded in the "real world," all of the possibilities of what finance *is*, with one exception, are necessarily platonic, requiring something that exists somewhere else outside the real world. This is of course an unusual condition for a "social" science. What we need is a truly "behavioral" finance that brings us back into the world again.

Philosophies of Mathematics

Finance models are mathematical. So if we are interested in what it means for them to be true or false, we must first consider what mathematical knowledge *is*, or what it means for mathematics to be true or false. There are generally two different perspectives on the nature of mathematics. None of the following dichotomies is identical to any of the others, and philosophers have proposed positions that combine entries from each column. But the terms in each column in Table 1 are largely consistent:

Table 1. *Perspectives on the Nature of Mathematics*

Platonic	Empiric
A Priori	A Posteriori
Analytic	Synthetic
Ideal	Natural
Infallible	Fallible
Permanent	Mutable
Necessity	Contingency
Discovery	Creation
Foundations	Social Constructions

Our discussion in this section will focus on the first and last perspectives. In Balaguer's [1998] terms, the platonic-empiric dichotomy addresses the *metaphysical project* (whether mathematical objects exist), and the foundations-social constructions dichotomy addresses the *hermeneutical project* (how to interpret mathematical theory and practice). Platonism is the view that there are mathematical objects (i.e., numbers, functions, sets) that exist outside of space-time. They are non-physical, non-mental, and acausal, existing independently of us, our theories concerning them, and our use of them (Balaguer [1998]).

Foundationalism is the view that all mathematical knowledge can be reconstructed in the form of a proof having a foundation of definitions, axioms, and rules of inference, and a superstructure of propositions or statements derived from them (Dancy and Sosa [1992], Ernest [1998]). If we accept the foundations, we must accept everything derived from them. The connection between the two projects is that foundationalism tacitly commits to necessary, infallible, a priori truths (Kitcher [1988]). So we accept the foundations because we believe they are true.

As can be inferred from its name, "platonism" is an ancient belief. In its later incarnation, it provided a philosophical justification for the origins of a modern science dependent upon mathematics. Galileo is reputed to have said that the laws of nature are written in the language of mathematics (Wigner [1960], Hamming [1980]). If there are laws of nature written, that must happen outside nature, or outside of space-time.

Today, platonism continues to be the dominant belief among mathematicians and scientists. This dominance is a consequence of a shared belief, expressed by the physicist Wigner [1960] in the title of his often-reprinted article "The Unreasonable Effectiveness of Mathematics in the Natural Sciences." Other famous physicists, such as Johannes Kepler, Heinrich Hertz, Richard Feynman, Roger Penrose, and Steven Weinberg, have expressed similar sentiments (Steiner [1989]).

Lakoff and Nunez [2000] summarize this point of view in what they call the romance of mathematics, and this myth may explain the attachment of finance to mathematical models. There is a strong desire for it to be true, and this desire is not wholly unselfish. Mathematical wizardry can indeed confer distinction and power (McGoun [1995]):

- Mathematics is an objective feature of the universe; mathematical objects are real, and mathematical truth is universal, absolute, and certain.
- What human beings believe about mathematics therefore has no effect on what mathematics really is. Mathematics would be the same were there no beings of any sort. Although mathematics is abstract and disembodied, it is real.

- Mathematicians are the ultimate scientists, discovering absolute truths not just about this physical universe but about any possible universe.
- Since logic itself can be formalized as mathematical logic, mathematics characterizes the very nature of rationality.
- Since rationality defines what is uniquely human, and since mathematics is the highest form of rationality, mathematical ability is the apex of human intellectual capacities. Mathematicians are therefore the ultimate experts on the nature of rationality itself.
- The mathematics of physics resides in physical phenomena—there are ellipses in the elliptical orbits of the planets, fractals in the fractal shapes of leaves and branches, logarithms in the logarithmic spirals of snails. This means that "the book of nature is written in mathematics." This implies that the language of mathematics is the language of nature, and only those who know mathematics can truly understand nature.
- Mathematics is the queen of the sciences. It defines precision. The ability to make mathematical models and perform mathematical calculations is what makes science what it is. As the highest science, mathematics applies to and takes precedence over all other sciences. Only mathematics itself can characterize the ultimate nature of mathematics (Lakoff and Nunez [2000, pp. 339–340]).

The philosopher Mark Steiner [1989] makes an even stronger argument for the "unreasonable effectiveness of mathematics" than the physicists. There is more to it than the theoretical predictions corresponding so precisely to empirical observations. There is also a correspondence between patterns of theory and patterns of observation. A successful prediction is simply a *first-order analogy*. Developments in physics have also arisen from *second-order mathematical analogies*, which are properties of descriptions (for example, linearity), and from *third-order mathematical analogies*, which are properties of properties of descriptions.

Another strong argument for platonism in the metaphysical project comes from the hermeneutical project. Even if mathematics appeared to be a social construction, we would not just have constructed it in order to use it successfully. Our refined ability to construct mathematics would only have evolved in a universe in which such an ability would have been selectively advantageous. If there were not laws of nature written in the language of mathematics, we would not have been able to construct the language with which to read them. And we would not in fact have *constructed* the language, we would have *discovered* it. In short, mathematics must be the loom upon which the fabric of the universe is woven, because if it weren't, we wouldn't have mathematics.

Although the social sciences, including finance, have not had such transcendental experiences with mathematics, platonism says they can, and social scientists' belief in it motivates them to use mathematics.

Contrary to the old saying, however, it *is* possible to argue with success. There are a number of explanations for the successes of mathematics that do not imply platonism (Wigner [1960], Hamming [1980], Steiner [1989], Dancy and Sosa [1992], Balaguer [1998], Lakoff and Nunez [2000]):

- Mathematics is successful at solving mathematical problems, and science is biased toward such problems. If a problem can be posed in mathematical terms, it is no surprise that it can be solved mathematically. Mathematics may even define what is and is not a problem and a solution.
- There is a lot of mathematics. Most of it will be unsuccessful at solving any given problem, and some of it has so far been unsuccessful at solving any problem outside mathematics. But of course our attention is directed to the mathematics that solves our problem. With so much to choose from, it is no surprise that some kind of mathematics can solve our problem.
- Some of the most fundamental concepts in mathematics, such as number, set, geometry, and logic, have alternative interpretations or versions that are ontologically distinct and incompatible. They cannot all be true of the same physical universe.
- Mathematics is merely convenient, not indispensable, for empirical science. Although mathematical expressions may be more efficient, theories can be nominalized, i.e., restated without reference to mathematical objects.
- There are many problems mathematics has not solved, nor is it likely to solve them. Mathematics may be successful in science because its use may have come to define what is and is not science. Finance and other "social" sciences behave as if they believe this, with subdisciplinary status being positively correlated with the use of mathematics.[7] Although uncommonly defined this way, the "humanities" may be called the repository for problems with which mathematics is unlikely to be especially successful.
- Mathematics is able to make what it is unable to do look as if it were still mathematics. In mathematics itself, what cannot be proven becomes an "axiom." In science, what cannot be predicted becomes an "initial condition." And in economics and finance models, we have "exogenous variables." Neither science nor mathematics can bootstrap themselves.

- Mathematics is no different than any other a priori, analytic, platonic concepts (i.e., definitions, classifications) that work for us. Such concepts only seem to be non-physical, non-mental, and acausal, existing independent of us, our theories concerning them, and our use of them. They are simply useful fictions—heuristic mental constructions that have facilitated our existence and may have even come to shape it. It is no coincidence that the symbolic structures of mathematics correspond to the natural structures surrounding us. If mathematics were truly platonic, we would not be able to know it (i.e., have causal interactions with acausal objects). The physical universe has no laws, only regularities. There can only be a fit within us between mathematical laws and the universe and not within the universe itself independent of us.

Most mathematicians, physicists, and other natural scientists believe in platonism, since it has a commonsense consistency with the "unreasonable effectiveness of mathematics" in their work. To repeat our earlier metaphor, if mathematics works so astonishingly well for them, it must be the loom upon which the fabric of the universe is woven.

It follows then that mathematics must hold the key to knowledge in the social sciences and to knowledge in the natural sciences. Therefore, its use by social scientists would seem to promise equally "unreasonable effectiveness." But as we have just shown, there are several excellent reasons not to attribute the "unreasonable effectiveness of mathematics in the natural sciences" to platonism, and there is no necessary reason (inherent in the warp or weft of the universe) to expect it in the social sciences. It may yet happen, but it doesn't have to.

Turning from the metaphysical project concerning the existence of mathematical objects to the hermeneutical project concerning the interpretation of mathematical theory and practice, we note there are three forms of foundationalism: logicism, formalism, and constructivism.[8] Logicism asserts that logical axioms and rules of inference alone are sufficient to derive all mathematics. Formalism says mathematics is a consistent uninterpreted formal system in which a set of definitions, axioms, and rules of inference are used to derive all mathematics. Constructivism believes mathematics is derived from axioms and rules of inference that we know intuitively to be true.

Recall from the definition of foundationalism that these all share a foundation of definitions, axioms, and rules of inference, as well as a superstructure of propositions or statements derived from them. They also share their reconstruction of mathematics in formal proofs, of which a comprehensive definition is worth quoting in full since similar proofs are the foundation of theoretical finance:

> The proof of a mathematical proposition is a finite sequence of statements ending in the given proposition, which sequence ideally satisfies the following property. Each statement is an axiom drawn from a previously stipulated set of axioms, or is derived by a rule of inference from one or more statements occurring earlier in the sequence. The term *set of axioms* should be understood broadly to include whatever statements are admitted into a proof without demonstration, including axioms, postulates, and definitions.
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> The idea underpinning the notion of proof is that of truth transmission. If the axioms adopted are taken to be true, and if the rules of inference infallibly transmit truth (i.e., true premises necessitate a true conclusion), then the theorem proved must also be true. There is an unbroken and undiminished flow of truth from the axioms transmitted through the proof to the conclusion.
>
> However, what has not yet been made clear are the grounds for the assumptions made in the proof. These are of two types: mathematical and logical assumptions. The mathematical assumptions used are the definitions and the axioms. The logical assumptions are the rules of inference used, which are part of the overall proof theory, as well as the underlying syntax of the formal language (Ernest [1998, pp. 6–7]).

We noted previously that foundationalism implicitly commits to necessary, infallible, a priori truths. This is clear for constructivism, which requires that there be a priori truths that we can intuit. It is less clear for logicism, however. Logicism had to retreat from its assertion that mathematics is wholly a branch of pure logic, without which no thought is possible, when it was discovered that mathematics also requires non-logical axioms and rules of inference (i.e., induction). It is still less clear for formalism, which as an *uninterpreted* system eschews truth as correspondence and concerns only truth as coherence.

Whether constructivist, logicist, or formalist, however, mathematics must be *about* something that can be reconstructed in the form of a foundation and superstructure that has to do with truth. Implicit in foundationalism is that this something is platonic and its truth a priori (Kitcher and Aspray [1988]).

Constructivism is consistent in its a priorism, but logicism and formalism are not. Although we know their foundations are fallible, that is, the definitions and axioms making up their foundations are no more than something we have agreed upon, the mathematical superstructures derived from them by the correct application of the proof method are believed to be infallible. But our agreement on the current foundations may not mean that we could have agreed on others; it may only mean that, since we cannot prove something out of nothing, we had to agree on something to begin with.

To intuit a foundation of a priori truths as in constructivism or to agree on a foundation of commonsense truths as in logicism and foundationalism may be an irrelevant distinction. Although superstructures have been built upon foundations of apparent falsehoods, such formal systems are not part of the mainstream of mathematics.[9]

All forms of foundationalism believe the proof method described in the quotation above is infallible in the transmission to the superstructure of whatever truth there is in the foundation. For logicism, it is the rules of deductive logic; for constructivism, it is intuitive rules of inference; for formalism, it is the rules of inference given for the system. The proof method, however, is as much an empirical matter as the axioms from which it must begin. It is very common for errors to be discovered in the informal proofs most common in mathematics. In addition, the definitions of a proof and of an error change over time. Few informal proofs have been formalized, some informal proofs cannot be formalized, and it is not possible to completely check a formal proof for errors. The method of proof itself has thus far been fallible (Ernest [1998]).

Finance models are in the form of mathematical statements constructed upon a foundation of assumptions by the same proof method used in mathematics. As we suggested in the first section, there is a very practical reason for this form. The explicitness of mathematical equations makes it easy to determine correspondence between predictions and outcomes (i.e., to justify truth), since something is or is not quantitatively equal to something else. But this does not explain why we require our models to be constructed as a formal mathematical proof.

If truth is an empirical matter, why is it not sufficient to just intuit a mathematical model? Why must we construct it in a way that transmits truth from a set of assumptions? We never test for correspondence between assumptions and observations, because we know there often is none. Recall that Friedman [1953] boldly stated that assumptions had to be false. We must usually make drastic simplifications in order to represent the world in a mathematically tractable way. But we want to construct true models, and the assumptions we choose are obviously not chosen arbitrarily. So we must believe there is some truth in them to transmit to our models. But then where does the truth in the assumptions come from, and how do we know it is there?

What we have discussed here about what mathematics is can also tell us something about those who do finance research. Along with the specific philosophy of platonism and the specific methodology of formalism come the values associated with platonism and foundationalism—"the abstract over the concrete, formal over informal, objective over subjective, justification over discovery, rationality over intuition, reason over emotion, generality over particularity, and theory over practice" (Ernest [1998, p. 269]). And it can also tell us something about the image of finance research that comes along with the mathematical form and content of its models.

> A widespread public perception of mathematics is that it is a difficult but completely exact science in which an elite cadre of mathematicians determines the unique and indubitably correct answers to mathematical problems and questions using arcane technical methods known only to them. This perception puts mathematics and mathematicians out of reach of common sense and reason, and into a domain of experts, to whose authority they are subject. Thus mathematics becomes an elitist subject of asserted authority beyond the challenge of the common citizen (Ernest [1998, pp. 273–274]).

Positive Finance?

Finance has traditionally interpreted its models more or less along the same lines as John Neville Keynes in his work originally published in 1891 (Keynes [1955]):

> As regards the scope of political economy, no question is more important, or in a way more difficult, than its true relation to practical problems. Does it treat of the actual or the ideal? Is it a positive science concerned exclusively with the investigation of uniformities, or is it an art having for its object the determination of practical rules of action? (Keynes [1955, p. 31])
>
> As the terms are here used, a *positive science* may be defined as a body of systematized knowledge concerning what is; a *normative* or *regulative science* as a body of systematized knowledge relating to criteria of what ought to be, and concerned therefore with the ideal as distinguished from the actual; an *art* as a system of rules for the attainment of a given end. The object of a positive science is the establishment of *uniformities*, of a normative science the determination of *ideals*, of an art the formulation of *precepts* (Keynes [1955, pp. 34–35]).

Machlup [1969] offers a variety of synonyms and an extended discussion of "positive science," "normative science," and "art" in his article, "Positive and Normative Economics: An Analysis of the Ideas." Both the terms "Positive" and "Normative" in his title have been used to convey a number of ideas, not all of which are perfectly consistent with Keynes' usage. Both Keynes and Machlup apply the term "normative" to the former, while Keynes refers to the latter as an "art," and Machlup as an "instrument."

As we assert, what finance *is*—positive, normative, artistic, instrumental, or something else—depends on what it means for its statements (models and hypotheses) to be true. Considering these historical perspectives on what economics/finance *is* along with the perspectives on the philosophies of mathematics described in the second section, the possibilities as we see them are:

- *Positive*—a statement is true if it corresponds to what people are doing (reflecting one variation of a correspondence theory of truth).
- *Normative*—a statement is true if it corresponds to what people should be doing (reflecting another variation of a correspondence theory of truth).
- *Instrumental/Pragmatic*—a statement is true if it corresponds to what would be useful for what people are doing (reflecting a pragmatic theory of truth).
- *Rhetorical/Metaphorical*—a statement communicates truth.
- *Foundational/Aesthetic*—a statement is true if it is a necessary consequence of a set of assumptions (reflecting a coherence theory of truth).
- *Social Constructive*—a statement is true if we agree it is true (reflecting a pragmatic theory of truth if what we agree on is useful).
- *Cultural*—truth is irrelevant.

The remainder of this section considers the positive interpretation, which is consistent with the traditional pattern of theoretical finance making predictions and empirical finance testing whether they come true. The fourth section examines the others.

The "positivism" of August Comte became the "positive economics" of Milton Friedman, which is still with us as the orthodox interpretation of finance. But what does "positive" really mean?[10]

> [Comte] used "positive" to refer to the actual in contrast to the merely imaginary, what can claim certainty in contrast to the undecided, the exact in contrast to the indefinite, the useful in contrast to the vain, and finally, what claims relative validity in contrast to the absolute (Habermas [1971, p. 74]).

With the positive interpretation, we must make statements, which can be either true or false, and compare their predictions to our observations. If there is a correspondence, the statement has been verified; if not, it has been falsified. The empirical evidence, the "facts," speak for themselves. This is the scientific method.

The correspondence theory of truth is itself controversial, as is the use of statistical methods to determine correspondence. But we are interested here in the use of the proof method to derive the models that we test in finance. Why are such models, derived by applying rules of inference to "false" assumptions, superior to more casual conjectures that we might make in the course of our informal experiences with finance practice? Why can't we just intuit our models as bold Popperian conjectures (Popper [1968])?

One reason is that we are looking for more than true statements—we are looking for foundations. This is consistent with a concern for explanation along with prediction. The modern philosophy of explanation began with the deductive-nomological model in Hempel and Oppenheim's [1953] paper "Studies in the Logic of Explanation." According to this model, an explanation is a deductive argument in which a description of the empirical phenomenon to be explained (explanandum) can be deduced from statements of conditions and general laws (explanans). Symmetry of explanation and prediction are consequences of the form of the argument. If the phenomenon has occurred, the conditions and the law provide an explanation; if it has not, they permit a prediction. While by definition every positive statement, wherever it comes from, makes a prediction, the proof method, as an extended deductive argument, makes the assumptions an explanation.

But there is some sense in which explanation and prediction are not symmetrical. Explanation seems to require "something more," that is, an answer to the "why" question that prediction does not require. An early candidate for this "something more" was causation. Now, in Salmon's colorful words, causality is a "hoary philosophical chestnut on which much ink has been spilled" (Salmon [1989).

But it is possible to deal with the issue in a relevant manner without delving into the deeper philosophical issues. The concept of causality is bound up in our perceptions of the relationships in our environment and in the ways in which we manipulate that environment in order to survive in it. The "causal-mechanical" approach to explanation is that the explanation of an event describes the underlying mechanisms that caused the event. The proof method makes the assumptions the "causes" of the derived "effects," requiring the rules of inference to be analogous to mechanical processes. Truth transmission is analogous to causation.

The "unification" approach to explanation introduces the issue of "understanding," which is another candidate for the "something more."

> This is the essence of scientific explanation—science increases our understanding of the world by reducing the total number of independent phenomena that we have to accept as ultimate or given. A world with fewer independent phenomena is, other things equal, more comprehensible than one with more (Friedman [1974, p. 15]).

In the unification approach, we begin our explanations with the most general applicable law (set of assumptions?) and become more specific (use the proof method?) to arrive at (derive?) the statements that correspond to an individual event. To explain a large number of phenomena as necessary consequences of a small set of assumptions is to do science.

So if we are seeking "something more" than just prediction, the proof method offers a foundation of assumptions as logical explanations (the deductive-nomological model), causes (the causal-mechanical model), or unifying concepts (the unification model). This *may* justify it as a source of hypotheses and models for empirical testing. But is it really the source, or is it a reconstruction that provides some sort of rationale for what we already intend to test? And what justifies our choice of assumptions?

If we eschew testing arbitrary conjectures about finance practice, what makes our assumptions concerning the fundamental determinants (explanations, causes, or unifying principles) of finance any less arbitrary? Does the fact that they are used as foundations for the proof method in model-building make them actual foundations? Is there a reason why rational behavior and perfect markets form the foundations for finance, other than our wanting them to be the foundations? Since many of our assumptions are empirically testable, why do we not search for more primitive foundations, of which they themselves are a derived superstructure? And why has the use of behavioral assumptions other than rationality met with such resistance (Fama [1998])?

The most intriguing issue is what it means for our models to be verified by our empirical tests. Logically, of course, neither verification nor falsification can mean anything necessary for the assumptions, whatever they are or wherever they have come from. But in another sense, if the model were falsified, the likely reaction would be disappointment, and some sort of action would need to be taken, usually making the assumptions more realistic and constructing and testing another model. But what usually happens is that the falsification process is challenged, and the model itself remains sacrosanct.

If the model were verified, the likely reaction would be to congratulate ourselves, at least provisionally, on our successful model-building. But then we would need to determine how such a correspondence could come about. An effectiveness of mathematics in finance would be magnitudes more unreasonable than in the natural sciences.

Suppose we convincingly demonstrate that options had been priced in accordance with the Black-Scholes option pricing model (hereafter B-S) before the model was constructed.[11] There are only two ways this could happen: 1) the pricing models options traders are using correspond to B-S, and they know it. Their models may not look like B-S, but there would be a straightforward translation between their models and B-S because all would have to yield the same prices. After all, translating one language into another necessitates that the speakers of both can make statements that work in similar ways.

And if traders already had working pricing models, we would hardly expect them to rush to adopt B-S, once published, unless it were a much more efficient method. The traders would certainly know that prices were no different.[12] Similarly, we would hardly expect someone who speaks one language to rush to begin speaking another unless it was much more efficient for some reason. And the person would certainly be aware that they were saying the same thing in the new language that they would have said in the old one.

The other way this could happen is 2) the pricing models options traders are already using correspond to B-S, but they do not know it. However, the market would need to be sufficiently competitive that anyone using a model that did not correspond to B-S would not survive. Under this scenario, we might reasonably expect traders to rush to adopt B-S, once published, since they would be unaware that they already had corresponding working pricing models. And being unaware of their prior models, they could never discover that prices had not changed with the adoption of the B-S model.

However, such ignorance defies belief. It would be as if someone constructed a new language that everyone quickly began to speak without realizing they were speaking another language before, and that it would be possible to translate the old language into the new language. This certainly seems to preclude a positive interpretation of finance models, but only for financial markets. We may still find that finance models apply elsewhere.

Consider how successful applications of economic models are in biology. This is possible only because other species are in a position to behave in accordance with a model or die, without ever being aware they are behaving in accordance with a model. Natural selection simply favors the retention of whatever behavior is advantageous for survival (Heinrich [1979], Smith [1982]).[13] The Black-Scholes model, for example, can never be a positive model in a social science, where humans are conscious of their own behavior and of their own models. But it may still be a positive model in the natural sciences, wherever there are analogs to options.

Other Finance?

The third section has shown that the traditional positive interpretation of finance models poses a number of difficult, perhaps intractable, philosophical problems. What about the other interpretations?

Normative

The normative interpretation of finance models, that they are prescriptive, also requires a correspondence theory of truth. But it is a correspondence with what people *should* be doing to achieve their objectives. This requires us to be able to measure the extent to which our objectives are being attained, and to devise a way to optimize that measurement. Of course, even if we know and can measure what we want to accomplish, it is impossible to know whether we are accomplishing as much of it as possible.

The proof method, however, makes it possible to view finance models as normative in spite of this limitation. If rational behavior is our objective, and we begin with simple statements that are indubitably rational and derive our models using rules of inference that transmit truth, then the behavioral prescriptions implied by the resulting models must be perfectly rational. In effect, we can achieve correspondence with the truth without having to empirically test correspondence with anything. Our models must be what we should be doing if we are to be perfectly rational.

This is seductive reasoning, and it is an implicit part of the traditional view of finance and its models. Although we assert they are positive, we must believe they are normative, or they wouldn't be taught in university courses or executive seminars. Both the teachers and the students must feel that the teachers can tell the students something they need to know and that they don't already know.

But of course there are flaws in a normative interpretation, the most obvious being the specification of the environment in the assumptions. While our models may correspond to what we should be doing in the perfect markets we assume, what we should be doing to be rational in the imperfect markets in which we actually find ourselves may be quite different, especially since we cannot quantify those imperfections.

But do we even know whether our simple assumptions are indubitably rational, as they appear to be? Devlin [1997] draws a clear distinction between *logical* and *rational*, with the former resembling what finance has called *rational*, and the latter what it has called *irrational* (or *quasi-rational*) (Thaler [1991]). It would in fact be quite irrational to be perfectly logical, to behave in accordance with the assumptions of the mathematical models of finance.

For example, even if we can rationally determine what information is relevant to a decision, we must retrieve information that turns out to be irrelevant in order to determine that it *is* irrelevant. Without emotion, this process would never end.[14] We would be paralyzed (de Sousa [1987]). There are cases of brain lesions that damage emotions and cause just this sort of behavior (Damasio [1994]). In short, true rationality recognizes that logic cannot bootstrap itself, because something must always be "given," and admits an emotional component into the decision-making process.

Instrumental/Pragmatic

The instrumental or pragmatic interpretation of finance models is that that they might "work." In effect, this is a more moderate version of the normative interpretation. A finance model works if it gets us closer to achieving our objectives than whatever else we could be doing or even whatever else we are doing. The pragmatic theory of truth[15] underlying this interpretation can be empirically tested. Provided once again that we can measure the extent to which our objectives are being attained, we can simply compare the prescriptions of one model to those of another, or to what we are doing now, and determine what works best.

But, again, what justifies our choice of assumptions? Is there a reason why the rational behavior and perfect markets most commonly assumed in the derivation of models should form the foundations for finance practice? Most likely we believe that whatever works best will have something to do with what we call rational behavior, so we begin the proof method with assumptions that embody this rational behavior as we perceive it.

Of course, the questions we posed about the normative interpretation concerning the environment and the definition of rational behavior also apply here, but they matter less. Ultimately, models that work are the "true" models, until we come up with models that work better.[16]

It is common in finance to seek out market imperfections by comparing selected behavioral heuristics against more naive approaches as applied to historical data. But it is rare to compare in practice the performance of one model against another to specifically determine which works best, even though it should be possible. In finance textbooks, "positive" models that have been repeatedly falsified are often referred to as "useful." This sounds like an instrumental or pragmatic interpretation of the model. The model doesn't describe what people are doing, but states they would be better off if they acted in accordance with the model. But "useful" is probably not "instrumental/pragmatic." It is more likely indicative of a rhetorical or metaphorical interpretation.

Rhetorical/Metaphorical

A metaphor is usually thought of as a literary or rhetorical figure or trope. Because the metaphor that something *is* something else is not literally true, metaphors look unscientific. In fact, we usually regard them as being somewhat dramatic (like Salmon's calling "causation" a "hoary philosophical chestnut"). Lakoff and Johnson [1980] define metaphors as ways of expressing one experience in terms of another. Something does not have to *be* something else in order to tell us something that helps us understand it. In terms of scientific explanation, which we introduced in our discussion of the positive interpretation of finance models, metaphors are explanatory in accordance with the unification approach. When we can explain one experience in terms of another, we reduce the number of independent phenomena in the world. Metaphors are not arbitrary. While not literally true, they have a basis in experience and illuminate previously hidden truths (Hesse [1965]).

The rhetorical/metaphorical interpretation of finance models is that model-building can expand and communicate our knowledge and understanding. As we noted, finance defends its explicitly false assumptions by asserting they must be false to represent the world in a mathematically tractable way. The world of finance is not as simple as we assume in our models, but to think about a world so simple illuminates our more complex world. Models do not tell us how the world is or how it should be, but they still tell us something about it. In short, models are the way in which we talk about finance and relate it to our beliefs about rational behavior. This is the way in which finance models are "useful."

Of course, we must ask what it means for finance models to be "useful" metaphors or rhetorical devices. In a scientific sense, it must mean we are able to make true statements that we would not have been able to make without the finance models. Although the models themselves are literally neither true nor false, they must reveal something else that is, which of course must be interpreted and justified in some way. We can certainly interpret finance models rhetorically and/or metaphorically as tools that facilitate our conversations. For example, they can express relationships among variables that may not have been apparent in other forms of communication. But is this enough? Is it possible for the conversation to be an end in itself, or is it necessary to eventually address the nature of the content of the conversation?

We must also be careful of the consequences of rhetorical devices and metaphors for our conversations. Any language we use naturally limits what we think and how we think, and something can become what we call it.[17]

For example, the Black-Scholes option pricing model very likely created the categories of assets called "derivatives" or "contingent claims" that it is supposed to be able to price. Such categories did not exist in finance until years after the publication of the model. It is not impossible that the model is an equation expressive of a deep structural identity that can actually be used to price these assets, but it is highly unlikely. Rather, it is a metaphor reflective of a structural similarity that can increase our understanding of the pricing of assets that concern choice or commitment (McGoun and Zielonka [2000]). With overreliance upon the model, however, we run the risk of turning too many phenomena into options, and thinking that the only things that matter in option pricing are the things that appear in the model.

Foundational/Aesthetic

The foundational/aesthetic interpretation detaches finance theory from finance practice. It is a pure formal system using the proof method to derive statements from a set of axioms using the rules of inference. The axioms may look as if they refer to human behavior and that they can be true or false, but this is an illusion. The entire structure is simple, elegant, perhaps even beautiful, and it makes no direct reference to anything.

This does not mean there can be no connection between finance models and finance practice, however. To construct a finance model is to construct an existence proof (Weintraub [1985]). The process tells us that if there were a world in which the assumptions were true, the finance model derived from them would be true of that world. But no connection between finance theory and finance practice is necessary.

It is possible to extract normative (or at least instrumental) prescriptions from existence proofs. The objective of Axelrod's [1984] theory of cooperation was to discover the necessary conditions under which cooperative behavior would emerge (in effect, construct an existence proof of it), with the aim of recommending what actions would actually foster its development. In finance, some models of markets and institutions have been used in this way to develop public policy prescriptions.

But this foundational/aesthetic interpretation returns us to many of the questions we posed for the positive interpretation, especially, why has the use of behavioral assumptions other than rationality met with such resistance? In a purely formal system, the foundations are irrelevant. We should explore the implications of different behavioral assumptions simply because they are different, not because some may be more "realistic" than others. Perhaps aesthetics concerns more than just the beauty of the proofs, but the beauty of the assumptions themselves. There is something special about rationality. There are many ways to be irrational (or quasi-rational, or a-rational, to use less pejorative terms), but we believe there is only one way to be rational.

If we are using the model to determine the conditions that result in desired outcomes, it certainly makes sense to examine many different sets of such conditions to determine which are easiest to implement. This is one of the ideas underlying agent-based modeling, using computer simulations of complex systems to explore the characteristics of agents that will result in certain behaviors (Axelrod [1997]). Such simulation models, however, cannot really be interpreted as foundational or aesthetic, since they lack the structure of mathematical proofs and are less likely to be described as "beautiful" than "astonishing." They would perhaps best be interpreted as metaphorical models, as we described in the preceding subsection. In effect, they blur the distinction between theoretical and empirical research that has traditionally structured finance.

Social Constructive

In the philosophy of science, and in science and technology studies, "social constructivism" usually means that knowledge is a reflection of the social processes by which it was constructed and not the representation of reality. Less often, it has the much stronger meaning that in constructing knowledge, society also constructs reality (Sismondo [1996]).[18]

A marked peculiarity of finance among the sciences, however, is that its object of study is the behavior of a few thousand people—most of whom are aware of the science of finance. While everyone engages in financial activity of one sort or another, only the decisions of a relative few have a significant impact. And most of these few have been educated in finance and stay relatively informed about the changes in finance. The part of society that has an interest in finance constructs its own pragmatic truth. The findings of finance feed back on the external environment so that there becomes no *there* out there independent of the study of finance, and we are highly susceptible to self-fulfilling/self-defeating prophecies (Watzlawick [1984]).

When research constructs a plausible reason for behaving in a certain way, it provides sufficient cause for us to behave in that way. These behaviors then become imbedded in common sense and reinforce that behavior in the future. Stories compete for the attention of the finance public, and whichever story makes the most sense becomes what people act on. This then constitutes the explanation of behavior until another more plausible story comes along.

It is possible that the proof method building upon assumptions of rational behavior is what gives a model or story the plausibility necessary to change behavior. And even though the assumptions themselves may be implausible, the whole process is sufficiently convincing to the opinion leaders who count that the behavior ends up being adopted by others. In this view, nothing about models matters except whatever it is about them that convinces the opinion leaders to convince others to act in accordance with them. It is a purely social process.

Another possibility is that one can construct just about any model by choosing the appropriate assumptions; therefore, it is no surprise that our models work. If they did not work, we would not have constructed them. The model is no more than a sophisticated confirmation of what we already believe to be true.

Finance models are supposed to be independent of the behavior they are intended to explain. That is, they are neither a cause nor an effect of it. The social constructive interpretation is that finance models are part of the process, either as a cause (behavior conforms to the model), or an effect (the model conforms to behavior), or both (models and behavior evolve together). The interpretation extends beyond theoretical finance to empirical finance. Although the facts are supposed to speak for themselves when the models are tested, these "facts" are as much a social construct as the models themselves.

Ironically, finance and economics are well-aware of self-fulfilling and self-defeating prophesies and socially constructed realities. However, they do not seem to have realized that, in many instances, what we expect to be true has to be true,[19] and the reality we construct is the only one there is.

For example, a general expectation that the price of an asset will fall because it is believed to be overvalued will lead to the sales that result in the price falling, which confirms that the asset was "overvalued." Of course, this presumes there is a "real" value next to which something can be over- or undervalued. Keynes' beauty contest metaphor for markets comes to mind: Markets are beauty contests in which the goal is not to select the most beautiful contestant, but to select he contestant the other judges will think is the most beautiful. This also presumes there is "real" beauty that can differ from assessed beauty.

Cultural

A final interpretation of finance models is that their truth does not matter. A model is just something that plays a role in our academic and practitioner cultures. Our statistical procedures for justification are wholly inadequate for determining whether our models correspond to what people are doing, what people should be doing, or what would be useful for what people are doing. The proof method is the scholastic Latin of today. It is the embodiment of reason and logic, free of emotional associations, and it lends distinction to whatever model it creates and to whomever created it (McGoun [1995]).

What matters is how adept we are at using the proof method to create aesthetically beautiful models that people agree to use. This is what earns the model-builders promotion, tenure, lucrative chaired positions at prestigious universities, and perhaps even Nobel prizes. As we have already said, the use of mathematics is correlated with status in the social sciences. In effect, we don't reward truth; rather, truth is rewarded to the successful models in this cultural process.

This is consistent with Bruno Latour's [1987] concept of science as an undertaking by networks of scientists competing with each other for resources and recognition. Models are the medium through which the competition takes place. A model is strengthened when someone is convinced, by whatever means (including empirical), to believe in it. It is weakened when someone is led, again by whatever means (even the same empirical ones!), to challenge it.

Platonism in Finance

As finance uses mathematics' method of proof to construct, or at least reconstruct, its models, the philosophies of mathematics should also be able to tell us something about the nature of finance. As we discussed in the second section, there are two different perspectives on the nature of mathematics—platonic/foundational, and empiric/social-constructional. Finance has consciously adopted the foundationalism of mathematics in its construction of models, and would deny that they are mere "social" constructs. As a self-proclaimed positive science that "lets the 'facts' speak for themselves" in empirical tests, finance would be reluctant to admit that anything about it is platonic. It is a science of what "is."

But as discussed previously, there are as many as seven interpretations of what finance models are. In this section, we evaluate them in platonic/foundational versus empiric/social-constructional terms. The positive interpretation would appear to be the antithesis of platonism. If the model were empirically verified, however, the only answer to how such a correspondence could have come about is that the "rational behavior" upon which the model is based is an ideal form of behavior that we have unconsciously and unintentionally adopted. If we observe people behaving "rationally," that is, in accordance with our models, there must be either some internal compulsion for them to do so, or some external compulsion in the form of societal pressures for them to do so. Either way, there is something about the behavior that is outside us and a part of the way the world works.

The normative interpretation clearly regards rational behavior as an ideal standard to aspire to and our models as prescriptions of how to behave to achieve it. The instrumental/pragmatic interpretation makes more modest claims for the results, because it derives its recommendations from statements of rational behavior. However, its implications are more or less the same.

On the surface, the foundational/aesthetic interpretation says nothing. But as with mathematics, we must believe it has something to do with transmitting truth from axioms of rational behavior to models or we would not be doing it. And the rhetorical/metaphorical interpretation implies some understanding of rationality that is worth communicating.

Rational behavior is not platonic the way mathematical objects are: non-physical, non-mental, acausal, and existing outside of space-time, independent of us, our theories concerning them, and our use of them. But another term on that side of the dichotomy in Table 1 applies. Rational behavior is "analytic" in that the finance definition of "rationality" is a necessary consequence of the biological definition of "life" (McGoun [1999]). Wherever we find life in the universe we find rational behavior, because if something did not behave "rationally," we would not consider is alive.[20] Since we observe so much behavior among humans that does not appear "rational," however, finance's definition of rational behavior must be an "ideal," to use another term from Table 1.

It is indeed difficult to imagine a platonic object in a social science such as finance. Take the familiar assumptions of perfect markets and perfect information, which are certainly ideal and infallible. Although we can conceive of mathematical objects like geometrical points, lines, and planes as existing outside space-time and independent of us, our theories concerning them, and our use of them, this is not true of markets or of information of any kind. We cannot conceive of having any meaning without ourselves undertaking transactions in the markets or knowing the information.

Consider an important platonic object in finance that appears in the Black-Scholes option pricing model—instantaneous variance. This can only be interpreted as a measure of the distribution of values that a variable could have had at a given time (but of course it only had one). At any given time, we observe a sample of one drawn from this distribution. The remainder of the population must exist outside space-time. To deal with this problem empirically, finance assumes that the values we observe over time constitute a sample drawn from a single distribution, and we can measure and use the parameters of this distribution. But, once again, the population itself must exist outside space-time.

We find this same platonic object—a distribution of what could have happened but didn't—elsewhere in finance. Normative valuation models assert that they compute the "intrinsic" value of an asset, or what someone should pay for it. The intrinsic value is the present value of whatever will be received in the future to settle the claim represented by that asset. To know the intrinsic value is to know both the value of the settlement and the correct rate at which to discount that value to the present. Because it will occur in the future, the value of the settlement is uncertain, and this uncertainty should be reflected in the discount rate in the form of a risk premium.

To justify a normative model in retrospect, we should be able to compare its computed value to the value we know we should have paid at that earlier time once we observe what has occurred subsequently (some authors use the term "ex-post" value). The problem is that to be the correct discount rate for the revealed value of the settlement, the rate of return would have to reflect the risk we should have expected. In other words, a correct "ex-post" value would require us to know not only what occurred, but what could have occurred but didn't, risk being a consequence of what could occur. Such hypothetical events can only exist outside space-time.

And something similar is used in empirical tests of positive models. All tests compute statistical significance, the measure of how likely a sample that has observed properties is to have been drawn from a population having certain properties. For example, how likely are we to observe a positive value if the "real" value is zero? In such tests, we implicitly assume that what we see is a sample drawn from a population of what could have happened but didn't. Yet again, we have something existing outside space-time.

An especially problematic concept is the "risk-free rate." Any "rate" must be a social construct, because it is the price at which capital is exchanged in a market. The traditional components of a rate—compensation for deferring consumption, compensation for expected inflation, and compensation for the possibility that the realized value of an investment will differ from the expected value—could not exist without a person (value of deferred consumption, expectation) or society (market, inflation). "Intrinsic value," however, means that such attributes as the value of deferred consumption and expectations can be correct or incorrect. There must be a "real rate" somewhere out there.

The problem is that for there to be no risk, there must be perfect foresight with regard to some asset. But while it might be theoretically possible that information and markets be perfect, foresight never will be. There may be a parallel here between a geometrical point, for example, and foresight. Real points can gradually become smaller, but the dimensionless geometrical point is still platonic. Likewise, one can structure an investment with a gradually smaller probability of a different settlement from what has been contracted, but a riskless investment is never possible. A risk-free rate is platonic in the same way as a geometrical point, but how can a risk-free rate be platonic if it is necessarily personally and socially grounded? Does finance envision a platonic society of platonic people outside of us and our society?

So we conclude there may very well be platonic objects in finance. And there is much more in finance that corresponds to the perspective of which platonism is a part. Finance takes a foundationalist approach in which its models are constructed on the base of assumptions concerning rational behavior using the proof method. At the very least, finance believes ideal rational behavior is the fundamental source of knowledge. At most, the analytic concept of rational behavior may even be a necessary attribute of life. This is the only explanation for why finance is so reluctant to use different sets of assumptions to derive its positive or foundational models.

Most of finance's interpretations of its models, including all of the traditional ones, fall within the platonic/foundational perspective as it is applied to mathematics. Even the positive interpretation does not reveal the "objective reality" that is supposed to be out there. There is no such thing, and even if there were, we would never know it. Only the social constructive and cultural interpretations are wholly different. For honorable (social constructive) or not so honorable (cultural) reasons, we've created our own world of finance and must study it as such.

"Finance" (AD—the Academic Discipline) is inseparable from "finance" (SSP—the Social Structures and Processes concerning the storage and transfer of monetary resources). This is not simple reflexivity, in which what we say about others also applies to ourselves. The subjects being studied are also the ones doing the studying. To adapt a metaphor famous in philosophy, in finance, we are rebuilding the boat while we are out at sea in it.

Notes

1. It is also called "financial economics," but we use the simpler and more familiar term "finance" here.
2. In the remainder of the paper, the term "finance" will refer to "finance" (AD) unless otherwise qualified.
3. When we refer to "models," we usually precede with the definite article "the"; when we refer to "theories" we do not. One notable exception is "the" arbitrage pricing theory, which has an equation and uses the definite article, thus appearing to be a "model." But as it incorporates the capital asset pricing model as a special case, it could be a "theory" within conventional usage, albeit a more limited one than most finance "theories." On the other hand, however, the Gordon growth model is a special case of a discounted cash flow "model"—we never refer to "the" discounted cash flow "theory."
4. Finance does not seem to have any laws of its own. The law of one price is from economics. Finance also employs the law of large numbers from statistics.
5. Arbitrage pricing theory is a "model" in that it may or may not be true with no further caveats.
6. Longino [1990] discusses how our values have a strong influence on the character of our research.
7. See McGoun [1995] for a description of the social effects of mathematics in economics.
8. "Constructivism" and "social constructivism" are different philosophies. "Intuitionism" is a form of "constructivism."
9. Belief in the infallibility of the superstructure of mathematics persists in the face of problems other than the fallibility of its foundations. Logicism and constructivism do not yield the same mathematics. It is possible to use deductive logic to prove statements we cannot intuit, but it is also possible to intuit the

truth of statements we cannot prove. We can intuit that the statement "This statement is unprovable" is true. Formalism requires an infallible meta-mathematics within the formal system to prove the consistency of the system. However, Gödel proved that to prove the consistency of a system requires a consistent meta-mathematical system making assumptions not in the system whose consistency is to be proven. This leads to an infinite regress.

10. We try not to fall prey to Hutchison's observation that "the term 'positivism' has become a 'kind of dustbin into which anything considered objectionable is summarily swept' " (Blaug [1994, p. 133, footnote 16], quoting Terrence Hutchison, *Changing Aims in Economics*).
11. If options are priced in accordance with the model after it has been constructed, it poses a different problem, which we address in the next section.
12. The Black-Scholes model was published shortly after the opening of the first specialized options exchange. Within months, traders were consulting the B-S tables of calculations of options values. The chronology is discussed in the *Economist* article "Of Butterflies and Condors" [1991]. Prior to the publication of B-S, traders were using tables of Sprenkle's "unknown parameters," for which B-S supplied values.

    Black and Scholes' initial article was submitted to the *Journal of Political Economy* on November 11, 1970. The final version was received by the journal May 9, 1972. It was published in the May/June 1973 issue (Black and Scholes [1973]). Financial folklore has it that the authors traded on the basis of their formula for some time before publication of their work. If their options pricing model were a positive model, this would not have been profitable. So it is unlikely they were committed to a positive interpretation of their model.
13. In the preface to his book applying game theory to biological problems, Smith [1982] explains why "game theory is more readily applied to biology than to the field of economic behavior for which it was originally designed" (p. vii). In addition to Darwinian fitness being more easily observed than utility, "the concept of human rationality is replaced by that of evolutionary stability. The advantage here is that there are good theoretical reasons to expect populations to evolve to stable states, whereas there are grounds for doubting whether human beings always behave rationally" (p. vii).
14. When there are costs associated with optimizing, we face the problem of optimizing our process of optimizing, and so on, *ad infinitum*. Economics refers to this as the "regress issue," and the literature concerning it is summarized in Conlisk [1996].
15. "The meaning of a concept is to be sought in the experiential or practical consequences of its application" (Dancy and Sosa [1992, p. 351]).
16. This is an admittedly extreme version of truth. A more moderate version is that a true model is the best one we will be able to attain in scientific practice, acknowledging we will never know when we have it. This admits and accepts a problem that the normative perspective dodges.
17. A strong argument against the correspondence theory of truth is that we cannot compare something to "objective reality"; rather, we are forced to compare it to our linguistic descriptions of what we perceive to be "objective reality," which is quite different.
18. "Social constructivism" can also refer to any product of society. In this weak sense, all of finance is a social construct.
19. This sounds a little like the assumption of rational expectations, by which the expectations in our models equal the eventual realizations. It is not the same phenomenon, however, since rational expectations are perfect forecasts of effects, not the causes of those effects.
20. This is a stronger assertion than the one we made in the second section. There we claimed that models of rational behavior were successful in biology because if species (except humans) did not behave "rationally," they would die.

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